Spectral Properties of Weakly Inhomogeneous BCS-Models in Different Representations

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For a class of Bardeen-Cooper-Schrieffer (BCS)-models, with complex, weakly momentum dependent interaction coefficients, the representation dependent effective Hamiltonians and their spectra are reconsidered in order to obtain a consistent physical picture by means of operator algebraic methods. The starting point is the limiting dynamics, the existence of which had been proved in a previous work, in terms of a C*–dynamical system acting in a classically extended, electronic Canonical Anticommutation Relations (CAR)-algebra. The C*-algebraic KMS–theory, including the low temperature limit, specifies the order parameters. These appear as classical observables, which commute with all other observables, constituting elements of the center of the algebra. The algebraic spectral theory, in the sense of Arveson, is first applied to the dynamics in general pure energy state representations. The spectra of the finite temperature representations are analyzed, identifying the gap as the lowest of those energy values, which are stable under local perturbations. Further insights are obtained by decomposing the thermal dynamical systems into the pure energy state Heisenberg dynamics, after having first extended them to more comprehensive W*-dynamical systems. The decomposing orthogonal measure is transferred to the infinite product space of quasi-particle occupation numbers and its support is characterized in terms of 0-1-laws leading to an asymptotic ratio of quasi-particles and holes, which depends on the temperature. This ratio is connected with an algebraic invariant of the representation dependent observable algebra. Energy renormalization aspects and pair occupation probabilities are discussed. The latter reveal, beside other things, the difference between macroscopic term occupation and coherent macroscopic term occupation for a condensate.

Key words: Superconductivity; Phase Transitions; Energy Renormalization; Arveson Spectrum.